Field Method for Estimating Soil Parameters for Nonlinear Dynamic Analysis of Single Piles

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Use of in situ soil properties increases the reliability and accuracy of numerical predictions. The problem of interest here is the nonlinear dynamic behavior of pile foundations. It is shown in this paper that soil parameters needed for simplified dynamic analysis of a single pile may be back-calculated from the dynamic response of the pile measured in the field. A pile was excited by applying a large horizontal dynamic force at the pile-head level, and the response measured. In this paper, two different (simplified) methods of modeling the dynamic response of the pile are considered. One of the methods is based on the Winkler foundation approach, with the spring constant characterized by the so-called nonlinear p-y springs. The second method is based on the equivalent linear finite element approach, with the nonlinearity of shear modulus and damping accounted for by employing the so-called degradation relationships. In the latter, the effect of interface nonlinearity is also considered. Starting with best estimates of soil parameters, the experimental data on the response of pile is used to fine-tune the values of the parameters, and thereby, to estimate parameters that are representative of in situ soil conditions. The soil parameters calibrated by the method can be applied to earthquake problems when the pore pressure build-up due to free-field response is not very high.

KEYWORDS piles, dynamics, soil, soil-pile system, soil-structure interaction, finite element method, Winkler method, nonlinear analysis.

INTRODUCTION

The behavior of a deep foundation depends on a set of complex factors such as the nonlinear constitutive behavior of soils including the effect of pore water pressure, soil-pile-superstructure interaction including slip and separation at the pile-soil interface, characteristics of the loading, superstructure compliance, etc. When the amplitude of loading is large, most of these factors control the behavior. Accuracy and reliability of the predicted behavior depend not only on the analysis method employed, but also on the accuracy with which the model parameters (or soil properties) are determined. In cases

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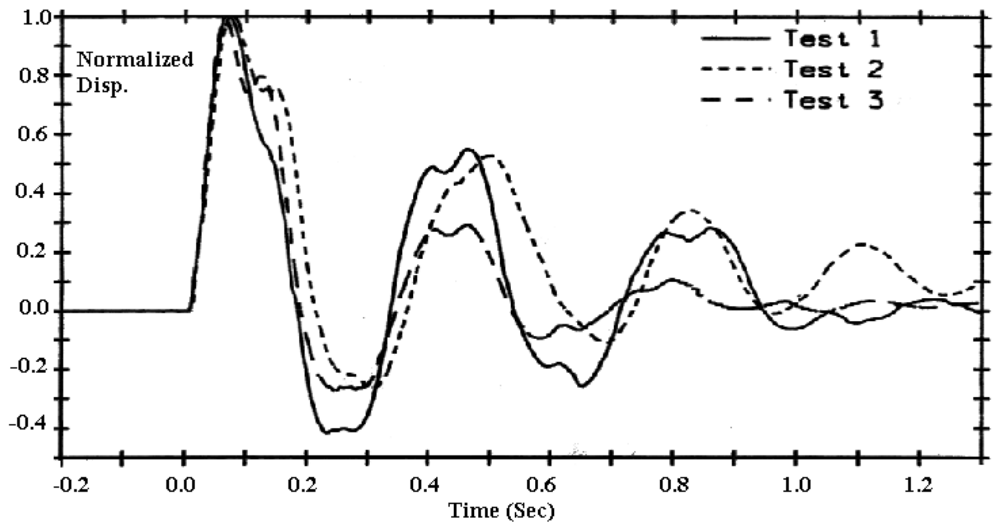
where good quality undisturbed samples can be obtained, most of the required properties can be determined by testing them in a laboratory. Due to unavoidable sample disturbance during sampling, transportation, and preparation during testing, in situ methods are preferred over laboratory methods. However, heterogeneity of natural soil deposits and approximations implied in analysis methods cannot be compensated by any of the above measures. This is, in fact, the reason that the static capacity of piles are in most cases determined directly from pile load tests performed in the in situ soil. A similar approach is highly desirable in the design of piles subjected to dynamics loads. It is shown that parameters for simplified analysis methods such as the Winkler foundation method and the equivalent linear finite element method may be determined from measured response of the pile subjected to a large-amplitude dynamic load applied at the level of the pile head. Experiments were conducted in a uniform granular soil deposit, filled in a test pit available at the test site, Turner-Fairbank Highway Research center (TFHRC), Federal Highway Administration, McLean, Virginia.

Fig. 1. A Photograph of the Field Test Setup

EXPERIMENTS

The tests were conducted in a 20 feet deep (6.1m) pit with a plan area of $18' \times 18'$ (5.5m×5.5m). The pit was filled with a uniform sand in loose to medium dense state about an year prior to the time the tests were conducted (for a different purpose). During this time period, the sand was subjected to rain several times. At the time the tests were conducted, the water table was below the level of the pile tip. The sand within the depth of the pile was damp due to capillary action.

A 4-in (0.1m) diameter, 12.3-feet (3.75m) long, pipe pile was driven into the soil to a depth of 9.2 feet (2.8m), with an overhang of 3.1 feet (0.95m). A weight of 122 lbs (0.54 kN) was attached to the pile at the pile head. The loading was to be applied by the Statnamic device (Middendorp, et al., 1992). The Statnamic device produces a single-pulse, impact loading. In order to extract more cycles of vibrations from a single-pulse impact loading, a spring-mass oscillator was attached to the pile head, and the Statnamic load was applied in the horizontal direction to the pile at the pile-head level through this spring-mass oscillator. The test setup is shown in Fig. 1. As a shot is fired from the Statnamic device, the projectile latches onto the spring (which is attached to the pile head), and oscillates along with the pile head. Three tests were conducted, each time with



a spring of different spring constant.

Fig. 2. Comparison Normalized Ground-Level Displacement Versus Time Histories of the Pile From the Three Tests.

The load experienced by the pile head was measured directly by a load cell attached between the pile head and the excitation setup. The horizontal displacement response of the pile was measured using 2 LVDTs (Linear Variable Displacement Transformers), one attached at the pile-head level and the other at the ground level. In addition, the horizontal pile-head acceleration was measured with the aid of an accelerometer. Fig. 2 presents a comparison of the ground-level horizontal displacement-time histories of the pile from the three tests. The peak displacements from the 3 tests were 0.8in, 1.5in, and 0.9in (2.0cm, 3.8cm, and 2.3cm) respectively. Except for the differences in the amplitudes, the frequency responses are almost the same. Only the results of the first test (Test 1, which has a peak amplitude of 0.8in) are used in the subsequent discussions and analyses. The load-time history is presented below along with numerical results.

ANALYSES BY THE FINITE ELEMENT METHOD

Details of the Method

The analyses presented in this paper are performed using HOPDYNE (Anandarajah, 1990), which is a finite element computer program with capabilities to model soil (linear and nonlinear), foundations, superstructure (linear) and soil-structure problems. The primary features of HOPDYNE that are used in the present study are

- 8-noded solid isoparametric element to model the soil
- 2-noded beam bending element to model the pile
- 4-noded three-dimensional slip elements to model the soil-pile interface
- Equivalent-linear approach to account for the nonlinearity of the soil

Details of these features can be found in published literature, and will not be repeated here.

The soil properties needed for equivalent linear finite element analysis are (for each soil type): $G_{\mathbf{max}}$, $\beta_{\mathbf{min}}$, G versus γ_{eff} relation and β versus γ_{eff} relation. The pile is characterized in terms of I_{xx}, I_{yy}, J_{zz}, A_p, E and ν, where I_{xx}, I_{yy} and J_{zz} are the second moment of inertias about $x-$, $y-$ and $z-$axes respectively (with the pile axis taken along the $z-$axis), A_p is the cross sectional area of the pile, E is the Young's modulus and ν is the Poisson's ratio. In addition, mass densities of the soil and pile are also required. While the properties of the pile are known and fixed, the properties of the soil are to be back calculated. To initiate the iterative process, starting estimates are required. The empirical equation suggested by Seed and Idriss (1970) is used to obtain a starting estimate for $\hat{G}_{\mathbf{max}}$:

$$\hat{G}_{\mathbf{max}} = K\sigma_m^{1/2} \tag{1}$$

where K is a density-dependent constant, and σ_m is the mean normal pressure. As the value of the coefficient of earth pressure at rest is unknown, it is assumed to be 1.0. With this assumption, σ_m becomes equal to the vertical effective stress. For a loose sand, with σ_m expressed in lb/ft^2 (or psf), $K = 40{,}000$. The density of the sand was about 110 lb/ft^3 (17.3 kN/m^3). As the soil is homogeneous, and the water table is not within the depth of the pile, Eq. 1 becomes:

$$\hat{G}_{\mathbf{max}} = 40{,}000\,(110h)^{1/2} = 0.42\times10^6 h^{1/2} \text{ psf} \tag{2}$$

where h is the soil depth. The optimal value of $G_{\mathbf{max}}$ is then taken as

$$G_{\mathbf{max}} = F_1\hat{G}_{\mathbf{max}} \tag{3}$$

where F_1 is a multiplier, to be established iteratively by matching the experimental response to the theoretical response.

The value of $\beta_{\mathbf{min}}$ was found to have a negligible influence on the overall response. On this basis, a value of 0.005% is assumed in the analyses reported here.

The G versus γ_{eff} relation and β versus γ_{eff} relation are function of the soil types. As the soil type is known, the empirical relations proposed for this soil type (sand) by Seed and Idriss (1970) are used, and assumed to be fixed. Thus, the only parameter that is sought by the back-calculation process is F_1.

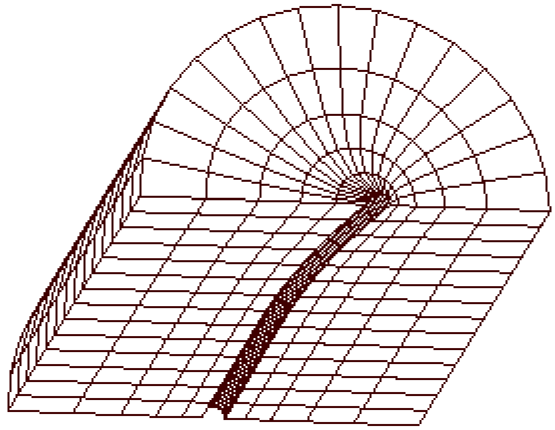


Fig. 3. Deformation of a Portion of the Domain Analyzed

Analysis and Results

The problem of interest is complex and three-dimensional. Three-dimensional finite element analyses can be computationally intensive, even in the case of a total stress based equivalent linear analysis such as the one of interest here. Equivalent linear analyses amount to a few (3 to 10) linear analyses. When pore water pressure effects are to be considered (which is the natural extension of total stress analyses), the computational efforts can be overwhelming to the point where the analyses can no longer be performed within a few minutes on a PC. Thus, from the point of view of using the analyses for practical design purposes, it is of interest to explore approximations that may yield results with acceptable accuracy.

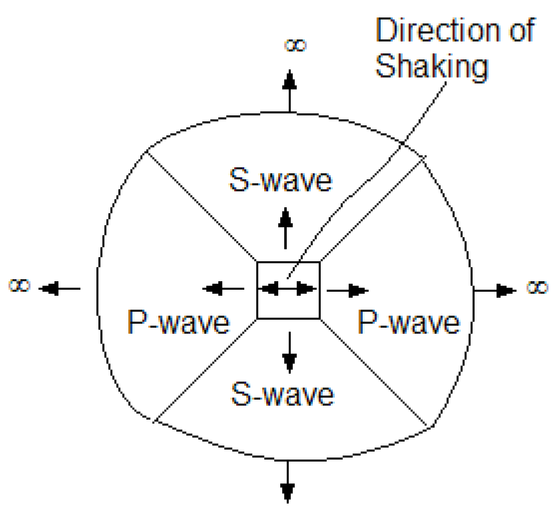


Fig. 4. Plain-Strain Approximation of Gazetas and Dobry (1984)

First, we consider a regular mesh with no approximations. As the problem is symmetric about the vertical plane that contains the pile and the direction of applied loading, only half the domain needs to be discretized. This cylindrical mesh, which contains 2824 nodes, 2240 soil elements, and 18 bending elements, provides accurate results for the problem. The outer boundary of the cylindrical domain is placed at a radius of 30 feet (9.15m). There are 14 layers of elements in the vertical direction. The deformation is pretty much confined to the region near the pile. To show the details a little better, deformation of the domain (at a given time) within a window around the pile is shown in Fig. 3.

Then two specific approximations are considered. In the past, several approximations have been developed for obtaining analytical solutions in the frequency domain (e.g., Gazetas and Dobry, 1984) and for approximate finite element solutions (e.g., Wu and Finn, 1997). We will use some of these as guidelines for our finite element analyses. In particular, Gazetas and Dobry (1984) developed models based on plain-strain approximations. In this, a slab of soil, extending to infinity in the radial direction, is assumed to have no displacements in the direction normal to the plane of the slab. At the center of the slab is a square-shaped pile segment undergoing a horizontal dynamic motion. The slab is divided into four quarters. As shown in Fig. 4, the energy that radiates away from the pile into the soil is assumed to take place in two distinct ways: (1) in the form of a compressional wave through two of the quarters, and (2) in the form of a shear wave through the other two quarters. It was shown that the radiation damping calculated from this approximate plane-strain model closely matched that radiation damping calculated from the plane strain solution of Novak, et al. (1978). The 14 plain-strain slabs, each with 4 quarters, are attached to the pile. In the radial direction, the slabs are fixed at 30 feet (0.915m) from the center. The slabs are not bonded in the vertical direction; i.e., each slab can undergo horizontal motions independently of each other.

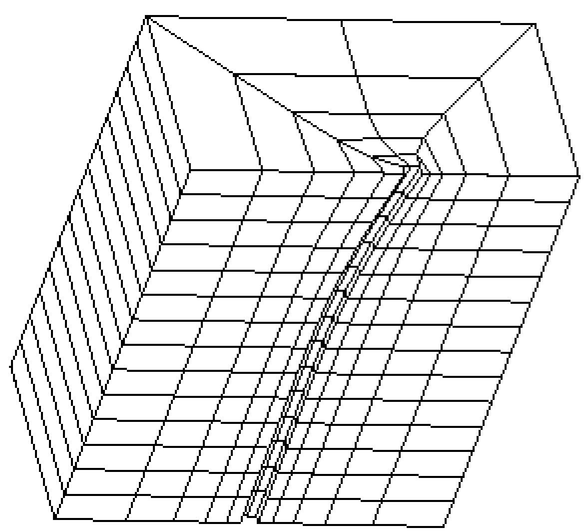


Fig. 5. Deformed Configuration Using Approximate, Coupled Mesh

The effect of bonding the plain-strain slabs in the vertical direction is examined using the mesh shown in Fig. 5, where, owing to symmetry, only half the domain is discretized. Primary difference between the coupled and uncoupled meshes are that the horizontal slabs are not bonded (i.e., uncoupled) to each other in the uncoupled mesh whereas they are in the coupled mesh. To further cut down the number of degrees of freedom, the vertical degree of freedom was suppressed in all of the analyses presented; its effect is examined next.

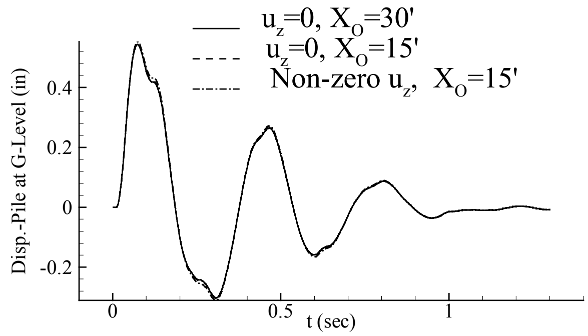


Fig. 6. Effect of Radial Domain Size and Vertical Degree of freedom

Fig. 6 presents a comparison of ground-level displacement-time histories obtained with the full mesh under the following conditions: (1) $X_0 = 30'$ and $u_z = 0$, (2) $X_0 = 15'$ and $u_z = 0$ and (3) $X_0 = 15'$ and $u_z \neq 0$, where X_0 is the radial distance at which a fixed vertical outer boundary is placed, and u_z's are the vertical degrees of freedom. It is seen that the differences in the results are very slight, indicating that the vertical response of the soil and the pile is negligible in the present case, and the radial distance of 30 feet is far enough to place a fixed vertical boundary.

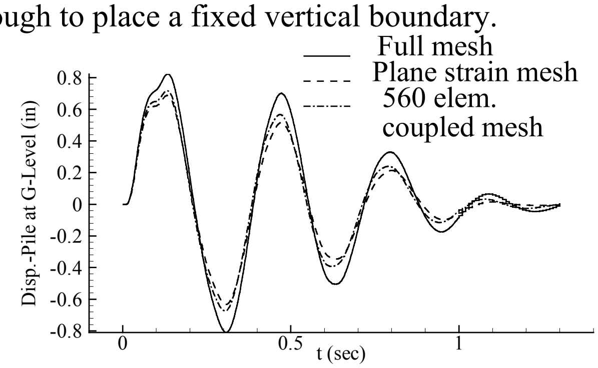


Fig. 7. Comparison of Results From Full, Uncoupled Plain-Strain and Coupled Meshes

Fig. 7 presents a comparison of ground-level displacement-time histories obtained with the three different meshes described earlier: (1) the full mesh, (2) the plain-strain mesh, and (3) the coupled mesh. While the results are not equal to each other, the approximate models appear to give acceptable results for practical use.

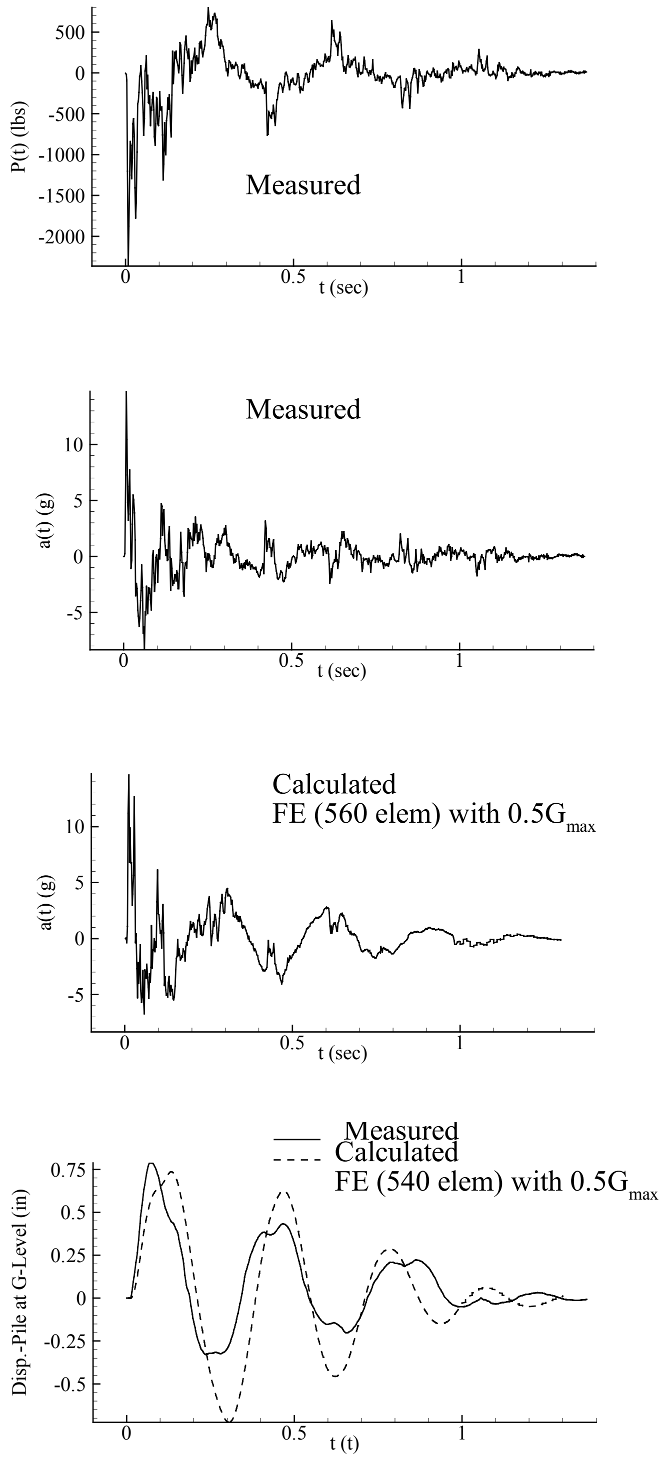


Fig. 8. Comparison of Calculated (Using Full Mesh and $0.15G_{\max}$) and Measured Results

It should be noted that these are not general results, and the outcome might differ from problem to problem, depending on the frequency of loading, natural frequencies of the system, etc. However, the results do indicate that for a given problem, it is worth exploring these approximations so that subsequent analyses (e.g., parametric study or a more systematic probabilistic study, where the analyses need to be repeated several times) may be performed using one of these approximate models.

After a few trial runs, a reasonably good match is found for $F_1 = 0.15$. A comparison between the finite element results (using the full mesh) with $F_1 = 0.15$ (Eq. 3) and the experimental results are shown in Fig. 8. The first plot in Fig. 8 presents the measured pile-head load versus time history, which is used as input to the finite element analysis. The 2nd and 3rd plots present the measured and calculated pile-head acceleration time histories respectively. The 4th plot presents a comparison between the measured and calculated ground-level displacement time histories. All of the above quantities are horizontal components.

In view of the fact that the soil is actually an elasto-plastic material, whereas it is represented by a form of a nonlinear viscoelastic model in the finite element analysis, the quantitative comparison shown in Fig. 8 is considered to be reasonably good. In both cases, the response dies out in about 3 cycles. The rate of decay of the displacement amplitude is predicted reasonably well.

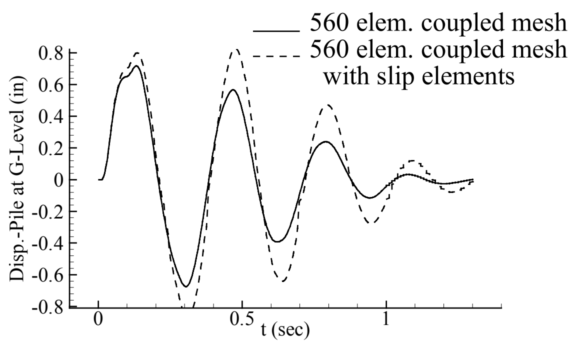


Fig. 9. Comparison of Calculated Responses with and without Slip Elements and $0.15G_{\mathbf{max}}$

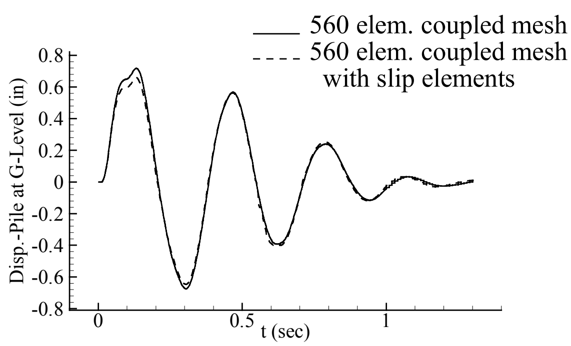


Fig. 10. Comparison of Calculated Responses with and without Slip Elements and $0.20G_{\mathbf{max}}$

In the analyses described so far, the soil has been bonded to the pile, preventing any gapping or slipping to take place at the soil-pile interface. Using the 560-element coupled mesh (Fig. 5), an analysis is conducted with slip elements placed between the pile elements and the soil elements. The calculated ground-level displacement time histories with and without the slip elements are shown in Fig. 9. It is noted that the use of slip elements renders the response softer, yielding larger pile displacements. These results were obtained with $F_1 = 0.15$. Then F_1 is varied until a good match is obtained. The numerical results with $F_1 = 0.20$ and with slip elements are compared in Fig. 10 with numerical results using $F_1 = 0.15$ and without slip elements. The results from the both analyses are virtually identical, indicating that the soil stiffness doesn't need to be reduced as much when slip elements are employed, since the use of slip elements makes the system softer by allowing slip and separation at the pile-soil interface. It should be noted that the consequence of allowing slip and separation may be more dramatic in other problems, and thus having the capability to use slip elements is desirable.

WINKER FOUNDATION METHOD

Details of the Method

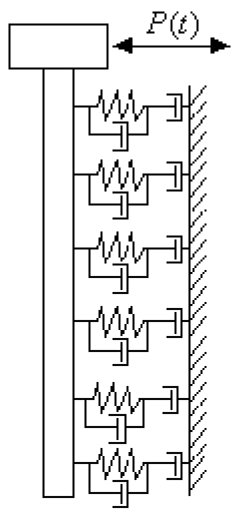


Fig. 11. Winkler Foundation Approach: Springs and Dashpots to Represent the Effect of Soil

In the Winkler foundation approach (Fig. 11), the primary member analyzed is the pile. The influence of the surrounding soil on the pile is introduced through a series of nonlinear springs (in the case of static problems). The most widely used spring force-displacement relationships are the so-called p-y curves of Matlock (1970) for clays and Reese, et al. (1974) for sands. In extending this static method to problems involving dynamic loads such as that from earthquakes, ship collisions, etc., methods are needed for accounting for damping – both material and radiation damping. Several researchers have worked on this problem (Kagawa, 1980; Berger, et al., 1977; Wang, et al., 1998; Boulanger at al., 1997; Loch et al., 1998; Badoni and Makris, 1996; Abghari and Chai, 1995, Nogami , et. al., 1992, Gazetas and Dobry, 1984, Sen et al., 1985; Trochanis, et al., 1991). These studies indicated that there are some major difficulties to be resolved.

Firstly, the most appropriate spring/dashpot model to use is not clear. For instance, Wang, et al. (1998), after comparing results with series (spring and dashpot in series) and parallel (spring and dashpot in parallel) models, determined that the parallel model leads to a very stiff system, with the damping force over-dominating the system response. In either case, the material damping needs to be considered as well. The appropriate model to use is thus yet to be identified.

Secondly, the issue concerning a suitable value to use for the coefficient of damping has not been resolved. For instance, if one uses Berger's model (1977) to represent the radiation damping, where it is assumed that radiation of energy away from the pile takes place in the form of p- and s-waves through a volume of soil of constant cross section (like a one-dimensional rod), the damping coefficient becomes frequency independent, and is given by

$$C = C_p + C_s = (V_p + V_s)\rho d \tag{4}$$

where V_p is the p-wave velocity and V_s is the shear (*s*-) wave velocity, ρ is the mass density and d is the diameter of the pile. Here C is the coefficient of damping per unit length of the pile. Wang et al. (1998) found that the value of C calculated using the above equation was too large. They arbitrarily assumed $C = 2V_s \rho d$. There is no consensus among researchers as to the most suitable model and the most appropriate equation for computing *C*. Berger's model is approximate. The manner in which the energy radiates away from the pile is complex, and the *p*- and *s*-wave portions cannot be easily separated out as is done in Berger's model. The cross section of the portion of the soil that carries the radiation energy is not constant. But analyses with non-uniform cross sections lead to frequency-dependent damping parameters (e.g., Gazetas and Dobry, 1984), making it difficult to apply them in a time-domain analysis. A frequency has to be arbitrarily selected for computing a value for C to use in a time-domain analysis such as that involved in the beam of nonlinear Winkler foundation method (BNWF). When the pile is shaken with a large amplitude loading – the problem of interest here - the material damping is more important than the radiation damping (Brown and O'Neill, 2001), and there is no rational method of calculating a value for the damping coefficient.

There are yet other factors that cannot be properly accounted for at the present time. For instance, the softening that takes place at the soil/pile interface due to slipping and gapping is difficult to model. Also, the pore water pressure build up during a cyclic loading such as the earthquake loading cannot be accurately modeled. All of the above difficulties associated with the use of nonlinear Winkler foundation methods point to the need for a site-specific, field calibration of the method.

In the specific Winkler foundation model used here, the soil is replaced by a series of elements involving a spring and a dashpot in parallel (i.e., without a second series dashpot shown in Fig. 11). The coefficient of damping is calculated according to Eq. 4, but with a modifier F_c, as follows:

$$C = F_c(C_p + C_s) = F_c(V_p + V_s)\rho d \tag{5}$$

Estimated values of V_p and V_s are 738 ft/s (225 m/s) and 492 ft/s (150 m/s), and the mass density is 3.42 lb-sec^2/ft^4 (1.76 kN-sec^2/m^4 or 1.76 g/cm^3).

The spring constant is represented by the p-y relation suggested by Reese, et al. (1974) for sands. Parameters of the p-y relation depend on whether the loading is static or cyclic; here the cyclic parameters are used. The entire curve is a function of (ϕ, γ', d, k_1), where ϕ is the friction angle of the soil, γ' is the effective density of the soil, d is the diameter of the pile, and k_1 is the slope of the initial straight line. The value recommended for k_1 for loose to medium dense sand is 60 lb/in^3 (16225 kN/m^3). Except for d, all other parameters can assume different values than our initial estimates. Let us, therefore, introduce multipliers with each parameter as:

$$\phi = F_\phi \phi^* \tag{6a}$$

$$\gamma' = F_\gamma \gamma'_* \tag{6b}$$

$$k_1 = F_k k_1^* \tag{6c}$$

The estimated value of the friction angle for this soil is 35^0, and the density is 110 lb/ft^3 (17.3 kN/m^3).

Analysis and Results

HOPDYNE (1990) has the capability to consider nonlinear, discrete springs and dampers. The beam bending elements available in HOPDYNE is used to model the pile.

The soil deposit is divided into 14 layers, and a parallel spring/dashpot elements is attached to the pile in the middle of each of these 14 layers. After several trials with different values for F_c, F_ϕ, F_γ and F_k, it is found that most matching results are obtained with $F_c = 1, F_\phi = 0.5, F_\gamma = 1$ and $F_k = 1$. In other words, changing only the value of ϕ not only is adequate, but gives the most optimal results. It may, however, be necessary to change some of the other parameters for best results in other problems.

The comparison between numerical and experimental results is presented in Fig. 12, where the plots at the top and middle are the measured and calculated pile-head horizontal acceleration-time histories, and the plot at the bottom is a comparison between numerical and experimental ground-level horizontal displacement-time histories. It is seen that the comparison is as good as the one with the finite element results (Fig. 8), indicating that both the equivalent linear finite element method and the beam of nonlinear Winkler foundation method are equally capable of representing the dynamic response of the single pile under study here.

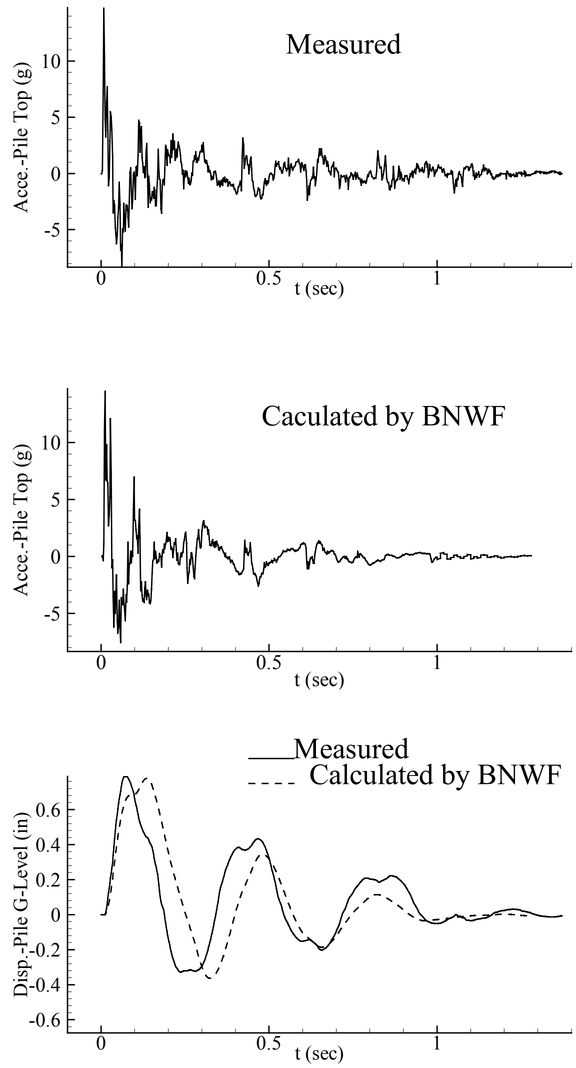


Fig. 12. Comparison Between Calculated (by BNWF Method with $F_{\phi} = 0.5$) and Measured Results

CONCLUSIONS

A series of large amplitude dynamic tests was conducted on a single pile driven into a homogeneous sandy deposit. The pile was subjected to a horizontal impact load with the aid of a Statnamic device. The pile underwent a cyclic motion involving about 3 cycles. The pile-soil system was modeled using two different numerical methods: (1) Equivalent linear finite element method, and (2) beam of nonlinear Winkler foundation method with nonlinear p-y curves to represent the stiffness and Berger's model to represent the damping. The objective was to back-calculate from the experimental results site-specific soil properties. The study indicates that both numerical methods are equally capable of representing the nonlinear dynamic response of the pile-soil system, and that the relevant soil parameters for these methods may indeed be back-calculated from the experimental data. While the beam of nonlinear Winkler foundation is simpler and computationally more efficient than the finite element method, the latter has the advantage of having the capacity to consider the interface slip and separation, and the effect of pore pressure (not considered in the present study).

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